

**Title: Similarity****Brief Overview:**

This unit develops the concept of similarity of polygons. It begins with an informal definition of similarity, defining them as figures with the same shape. Then, through exploring relationships among corresponding parts of similar figures, students are lead to a formal definition of similar polygons. This definition can be applied to analyzing similar triangles in different configurations. Students will test, through inductive reasoning, the three ways to prove triangles similar. Students will build a pantograph and apply their knowledge of similar triangles to calculate the scale factor of images they magnify with it.

**NCTM Content Standard/National Science Education Standard:**

Geometry, Measurement, and Reasoning

- Explore relationships (including congruence and similarity) among classes of two and three dimensional geometric objects; make conjectures about them; solve problems involving them.
- Draw and construct representations of two and three dimensional geometric objects using a variety of tools; use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.
- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

**Grade/Level:**

Grades 9 – 10, Geometry

**Duration/Length:**

Four 45-minute lessons

**Student Outcomes:**

Students will:

- Develop an understanding of the properties similar and congruent polygons
- Write similarity statements for similar triangles
- Use similarity to write ratios of corresponding sides
- Use given information to analyze figures with similar triangles
- Identify situations in which there is sufficient information to conclude that similar triangles exist
- Build a pantograph, describe the principle which makes it work and draw enlarged figures with it

## Materials and Resources:

- Rulers
- Protractors
- Scissors
- **Student Pantographs**
  - Large craft sticks with holes drilled,
  - Screws,
  - Brass fasteners,
  - Markers,
  - Mini suction cups
  - “Building and Using a Pantograph” instruction sheet
- Metal Pantograph
- Patty paper
- **TI-84** overhead calculator with **Cabri Jr** application
- **TI-84** overhead calculator screen viewer
- Overhead projector
- **Overhead Transparencies**
  - Similar Triangle Overhead
  - AA Similarity
  - SSS Similarity
  - SAS Similarity
- **Worksheets**
  - Similar Polygon Cut-Outs
  - Defining Similarity
  - Writing Similarity Statements and Using Proportions
  - Identifying Similar Triangles
  - Constructing a Similar Hexagon Design
  - Directions for Creating a Model of a Pantograph on CabriJr
  - Summative Assessment

## Development/Procedures:

### Lesson 1

Preassessment – Have students review concepts of equal ratios, classifying angles and triangles, and using a protractor to measure angles.

Launch – Begin the lesson by showing students simple real-life examples of similar objects, such as reduced and enlarged images from a photocopier and model cars versus actual cars. Introduce the word “similar” to mean two shapes that which have the same shape but not necessarily the same size. Hand out triangle cut-outs. (See similar polygon cut-out sheet.) Instruct the students to sort the triangles into piles of acute, obtuse, and right triangles. Check answers.

Teacher Facilitation/ Student Application – Present the purpose of the lesson, which is to come up with a definition for similar polygons based on relations among their parts. Ask the students, “Are all triangles similar?” Ask students to compare all the triangles. Ask students’ “Can a right triangle be similar to an obtuse triangle?” Direct students to find pairs of triangles that appear to be similar and to decide which triangles are not similar to the others.

Hand out the worksheet, “Defining Similarity.” Instruct the students to measure and record the angles of the similar triangles. Lead them to form conjectures by asking, “What appears to be true about the angles in similar triangles?” Repeat this discovery activity for the lengths of sides of the triangles. Have students measure and record the lengths of sides of the similar triangles on the worksheet. Before comparing ratios, check the students’ measurements to make sure they are all correct. Have students write ratios for the longest, middle, and shortest sides in each pair of triangles. Students should be able to recognize that the ratios are equal for the pair of equilateral triangles. To compare the ratios for the right and isosceles triangles, have students use a calculator to check that the ratios are equal.

Embedded Assessment – To assess students’ comprehension present a few questions, such as “Are all right triangles similar?”, “Are all squares similar?”, and “Are all rhombuses similar?” Ask students to provide explanation to support their answers, and to provide counterexamples when appropriate.

Reteaching/Extension –

- For those who have not completely understood the lesson, review what they need.

## Lesson 2

Preassessment – Have students review concept of similar triangles, and identify the part that will be congruent (angles) or proportional (sides) in any pair of congruent triangles.

Launch – Begin the lesson by showing students the Similar Triangle Overhead. Have students name the sides and angles of each triangle in order of appearance from largest to smallest. Ask students if the triangles appear similar (they do). Explain that they will need a formal way to express this fact.

Teacher Facilitation/ Student Application – Introduce the similarity statement  $\triangle ABC \sim \triangle XYZ$ . Point out that the corresponding angles appear in the same order in each triangle. Ask the students what should happen to the statement if the order of the letters in

$\triangle ABC$  is changed to  $\triangle BCA$ . (It should become  $\triangle BCA \sim \triangle YZX$ .) Ask the students to provide one additional statement for the same pair of triangles. (e.g.  $\triangle CBA \sim \triangle ZYX$ )

Add the following side lengths to the triangles on the transparency:  $AB = 5$ ,  $BC = 6$ ,  $AC = 9$ , and  $XY = 15$ . Remind the students that corresponding sides in similar triangles are proportional and explain that they can use this fact to determine the missing lengths of the sides of  $\triangle XYZ$ . Demonstrate writing the proportion

$\frac{AC}{XZ} = \frac{BC}{YZ} = \frac{AB}{XY}$ . Point out that students will have an easier time writing this proportion if they always keep the sides of one triangle on the top of each fraction and the sides of the other triangle on the bottom, and if they pair longest side with longest, middle with middle, and shortest with shortest. Model using the proportions to solve for the unknown sides of  $\triangle XYZ$ .

Embedded Assessment – Have students do the worksheet “Writing Similarity Statements and Using Proportions.”

Reteaching/Extension –

For those who have not completely understood the lesson, review what they need.

### Lesson 3

Preassessment – Review the definition of similarity. Using an example, ask, “What would you need to know in order to conclude that the two triangles are similar?” Facilitate arriving at the correct answer. Follow up by stating, “Today, our purpose is to find if triangles can be proven similar based on less information than all that the definition requires.”

Launch – Distribute three sheets of patty paper to each group. Ask student groups to use a protractor to draw and label three angles, one on each sheet of patty paper, that measure  $50^\circ$ ,  $60^\circ$ , and  $70^\circ$ . Confirm with them that the sum of these angles is  $180^\circ$ , so that they could be the angles of a triangle.

Teacher Facilitation/Student Application – Direct students to make a triangle with the three angles by overlapping the three sheets. Challenge students by asking, “Is it necessary to use the third angle? Is there more than one way to do this? Are all the possible triangles similar to each other? Will the triangles be congruent or similar?” Elicit responses. To support that the triangles are similar, have students trace two possible triangles and compare ratios of sides. Identify this as the AA Similarity Postulate.

Use the AA overhead to provide additional examples of using the AA similarity postulate.

Embedded Assessment– After the AA similarity postulate, extend student thinking by asking, “What is the purpose of the AA similarity postulate?” “Does it work for rectangles? If not, give a counterexample.” “Does it work for rhombuses? If not give a counterexample.”

Repeat this procedure with three sheets of patty paper for the SSS similarity postulate, except have students enlarge or reduce sides by a certain scale factor. This will probably require some review. Instruct students to put the sides together to form triangles. Have students name the segments  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$  for one triangle, and label the corresponding sides in the second triangle  $\overline{A'B'}$ ,  $\overline{B'C'}$ , and  $\overline{A'C'}$ . Ask if the corresponding angles are congruent and have students justify their responses. Explain how this connects the SSS Similarity Postulate to the AA Similarity Postulate.

Use the SSS overhead to provide additional examples of using the SSS similarity postulate.

As a class, discuss SAS Similarity. Draw an included angle between two sides of lengths 3 and 4. Complete the triangle. Then, lengthen each side by 50% and connect these to make a second triangle. Now that the triangles are complete, measure the corresponding angles. Ask students to justify that the triangles are similar using AA. Explain how this proves the SAS similarity theorem, and reinforce the SAS similarity theorem with examples.

Use the SAS overhead to provide additional examples of using the SAS similarity postulate.

Reteaching/Extension – Distribute practice activity, “Identifying Similar Triangles.” Let students work independently. Allow those who still do not comprehend some part of the lesson to work in pairs.

#### Lesson 4

Preassessment – Review a triangle with a segment connecting two sides and parallel to the third side.

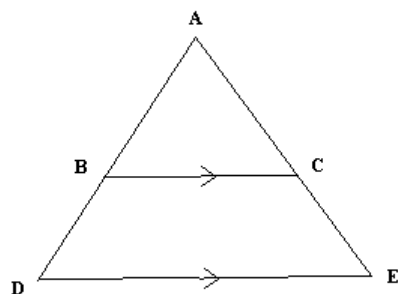


Figure 1

Discuss the three ways it can be concluded that the two triangles that appear are similar by AA, SSS, and SAS. Draw a segment that connects the endpoint of the parallel segment to the base so that a parallelogram is formed.

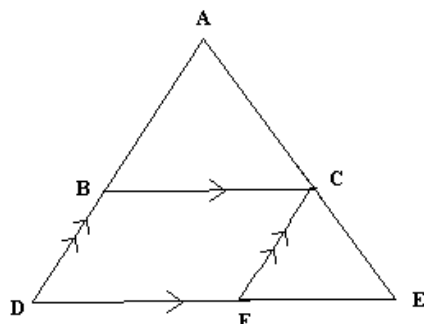


Figure 2:  $\triangle ABC \sim \triangle ADE$

- Launch – Model figure 2 on a TI-84 in the CabriJr application. To save time, this can be created and saved before class and shown as a demonstration to students. (See Directions for Creating a Model of a Pantograph on CabriJr.) Drag point C to show how all the triangles in the figure remain similar when the dimensions change. Point out that the parallelogram changes shape, but remains a parallelogram, and the similarity relationship is maintained. This figure is the basis for the design of a drawing tool called a pantograph. A pantograph is a tool that draws a magnified image.

Teacher Facilitation/Student Application – Hand out the supplies and instructions for the pantograph and have students put them together. Results may vary. Briefly demonstrate the use of a real pantograph on the board. Have students determine the scale factor of the drawing and how the tool is designed to create a drawing specifically with that scale.

Embedded Assessment – Monitor students' construction of the pantographs, giving tips on using it correctly.

Reteaching/Extension –

- Allow students to practice creating magnified images. If they do not see why it works, go back to the TI84 and point out how the parts of the pantograph relate to the figure.
- For students who master the pantograph, have them work on the “Constructing a Similar Hexagon Design” worksheet

**Summative Assessment:**

See summative assessment worksheet.

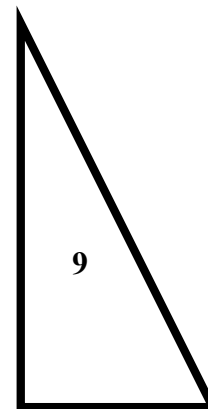
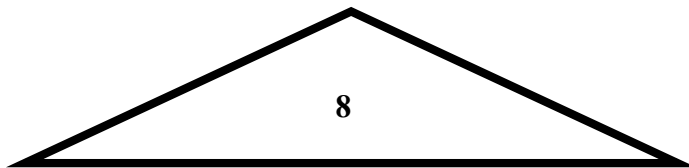
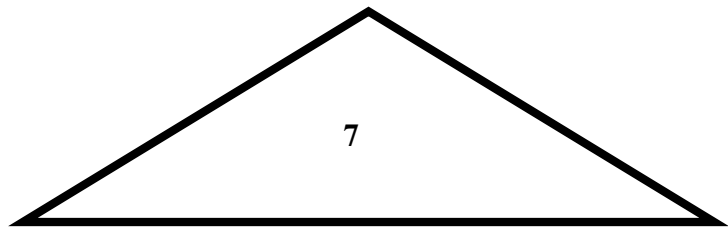
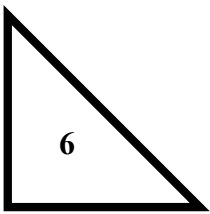
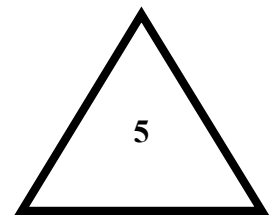
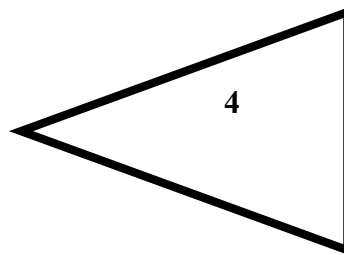
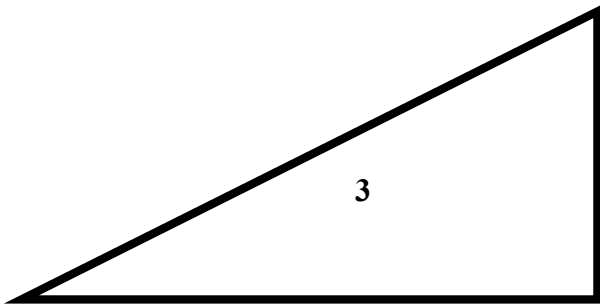
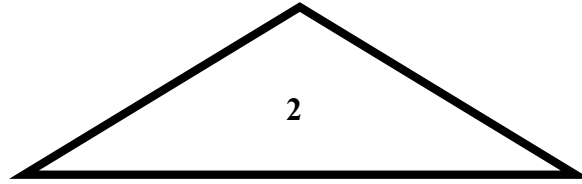
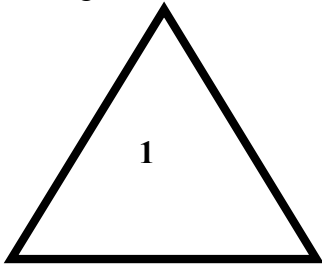
**Authors:**

Aimee Sann  
Bais Yaakov School for Girls  
Baltimore, Maryland

Matt Damseaux  
Baltimore City College High School  
Baltimore, Maryland

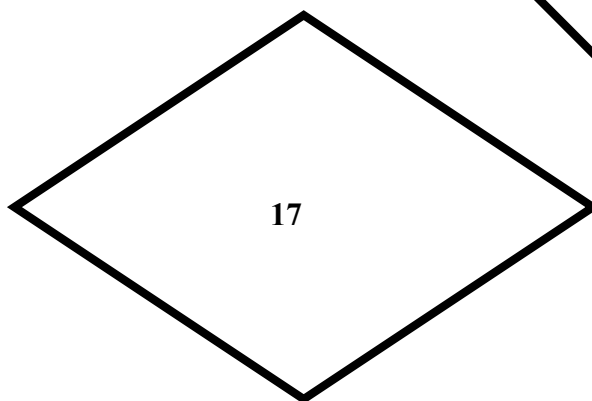
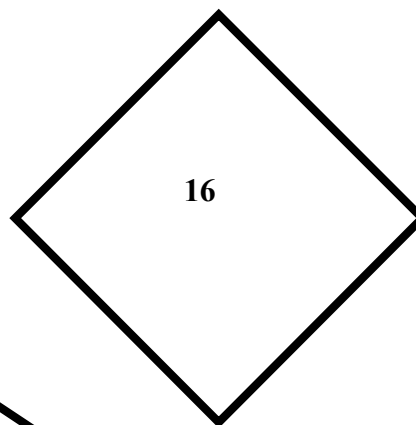
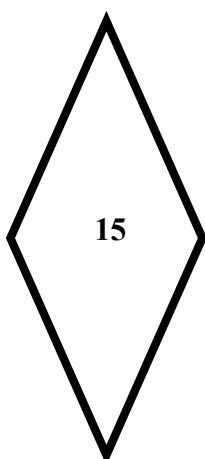
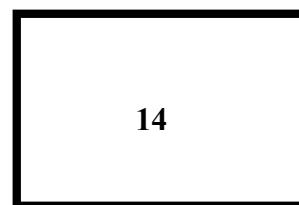
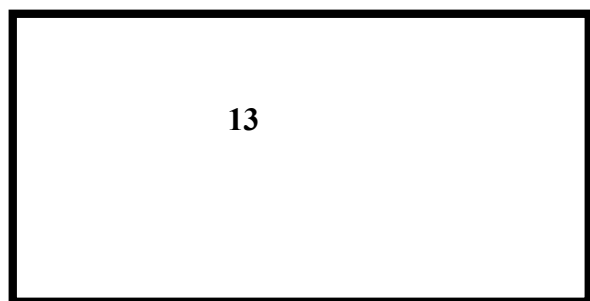
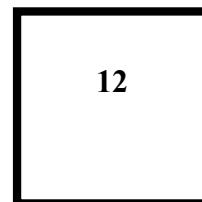
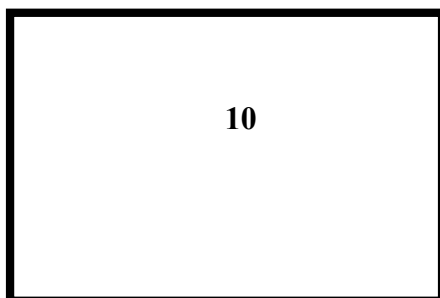
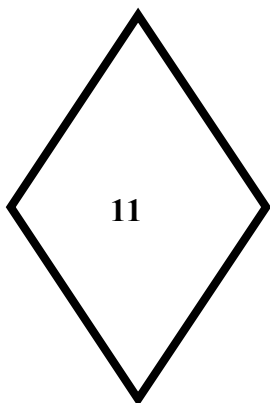
## Similar Polygon Cut Outs

Teacher: cut out each of the triangles below in advance, and distribute one set for each student. Direct students to sort the triangles into piles of acute, right, and obtuse triangles.





Teacher: cut out each of the quadrilaterals below in advance, and distribute one set for each student. Direct students to decide which pairs of quadrilaterals are similar.



## Defining Similarity

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Now its time to come up with a more formal definition for similarity.

For each of the pairs of triangles that you decided were similar, measure all sides and angles and record the results in the table below.

	Largest Angle	Middle Angle	Smallest Angle	Longest Side	Middle Side	Shortest Side
Triangle 1						
Triangle 5						
Triangle 3						
Triangle 9						
Triangle 2						
Triangle 7						

1. What do you notice about the measures of corresponding angles for each pair of similar triangles?

2. What do you notice about the ratios of the measures of the corresponding sides for each pair of similar triangles?

3. Use your observations to write a formal definition of similarity:

Two polygons are similar if and only if...

## Similarity Day 1, Worksheet 2: Defining Similarity Answer Key

Now its time to come up with a more formal definition for similarity.

For each of the pairs of triangles that you decided were similar, measure all sides and angles and record the results in the table below.

	Largest Angle	Middle Angle	Smallest Angle	Longest Side	Middle Side	Shortest Side
Triangle 1	60	60	60	4cm	4cm	4cm
Triangle 5	60	60	60	3cm	3cm	3cm
Triangle 3	90	64	26	8.3cm	7.4cm	3.7cm
Triangle 9	90	64	26	5.4cm	4.8cm	2.4cm
Triangle 2	120	30	30	7cm	4cm	4cm
Triangle 7	120	30	30	8.9cm	5.1cm	5.1cm

What do you notice about the measures of corresponding angles for each pair of similar triangles?

They're Congruent!

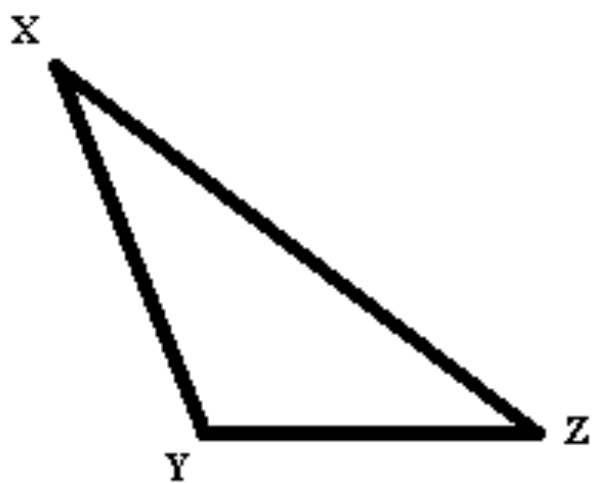
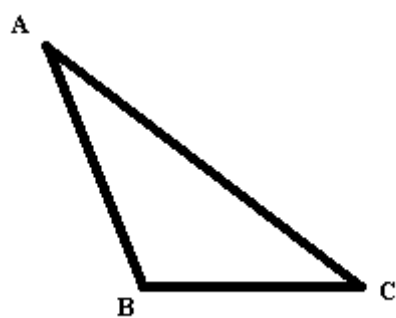
What do you notice about the ratios of the measures of the corresponding sides for each pair of similar triangles?

They're (pretty much) the same.

Use your observations to write a formal definition of similarity:

Two polygons are similar if and only if...all corresponding angles are congruent and all corresponding sides are proportional.

## Similar Triangle Overhead Transparency



# Writing Similarity Statements and

Name: \_\_\_\_\_

## Using Proportions

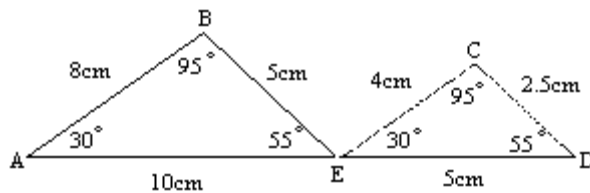
Date: \_\_\_\_\_

For exercises 1-3, write a similarity statement of each pair of triangles. Then write a

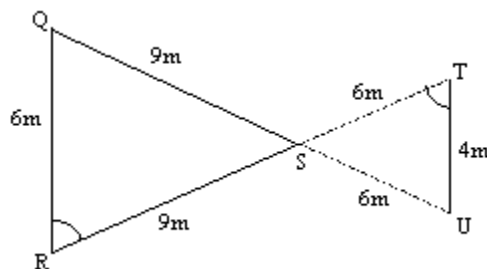
$\triangle ABE$   $\square\square$  \_\_\_\_\_ proportion for the corresponding sides.

$$\frac{AB}{EC} = \frac{BE}{ED} =$$

1.

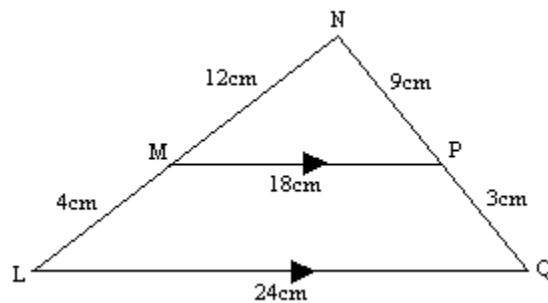


2.



$\triangle QRS$   $\square\square$  \_\_\_\_\_

3.



$\triangle MNP$   $\square\square$  \_\_\_\_\_

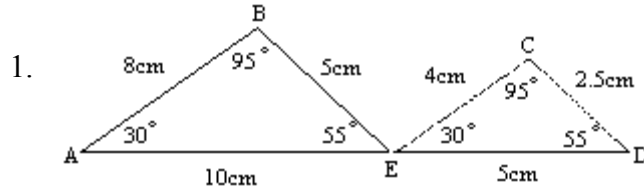
4.  $\triangle ABC \square\square \triangle TUV$ .  $\angle A \cong \angle$  \_\_\_\_\_?  $\frac{AB}{TU} = \frac{AC}{UV}$ ?

5.  $\triangle HAT \square\square \triangle PIN$ .  $HA = 2$ ,  $AT = 3$ , and  $PI = 5$ .  $IN =$  \_\_\_\_\_?

**Similarity Day 1, Worksheet 3: Writing similarity statements and using proportions.**

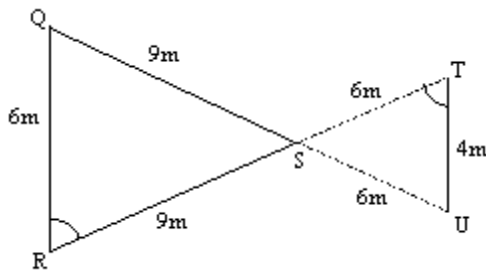
**Answer Key**

1-3: Write a similarity statement of each pair of triangles. Then write a proportion for the corresponding sides.



$$\triangle ABE \sim \triangle ECD$$

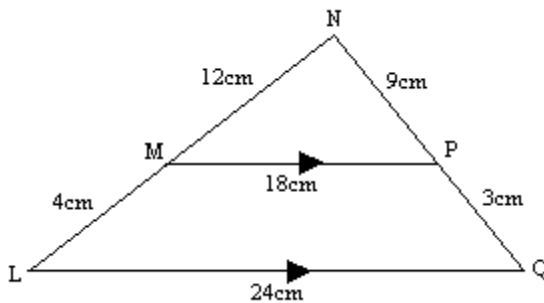
$$\frac{AB}{EC} = \frac{BE}{CD} = \frac{AE}{ED}$$



$$\triangle QRS \sim \triangle TUS$$

$$\frac{QR}{UT} = \frac{RS}{TS} = \frac{QS}{US}$$


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$$\triangle MNP \sim \triangle LNQ$$

$$\frac{MN}{LN} = \frac{NP}{NQ} = \frac{MP}{LQ}$$

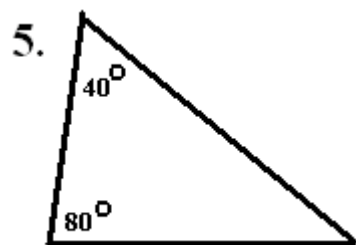
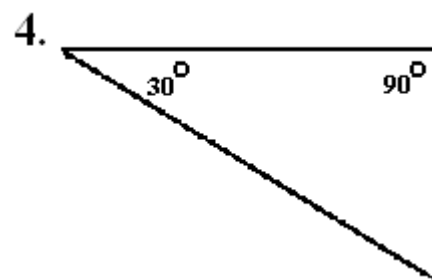
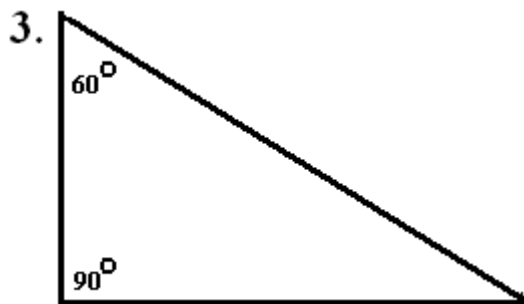
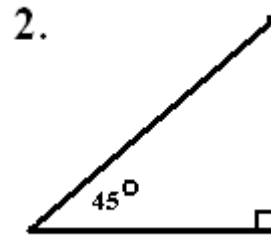
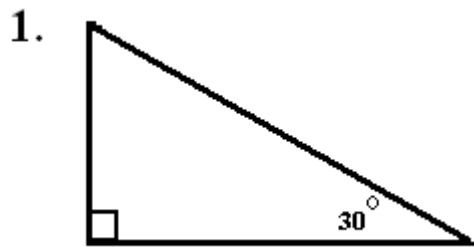

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4.  $\triangle ABC \sim \triangle TUV$ .  $\angle A \cong \angle T$   $\frac{AB}{TU} = \frac{AC}{TV}$

5.  $\triangle HAT \sim \triangle PIN$ .  $HA = 2$ ,  $AT = 3$ , and  $PI = 5$ .  $IN = 7.5$

## AA Similarity Overhead Transparency

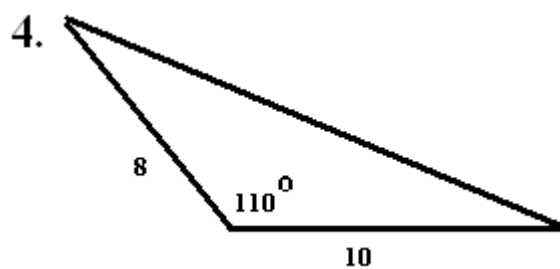
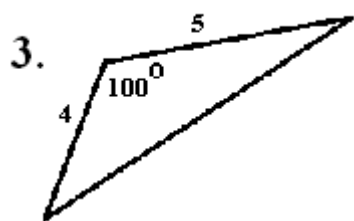
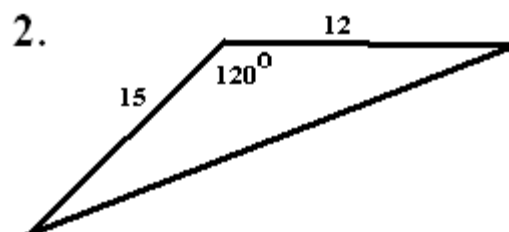
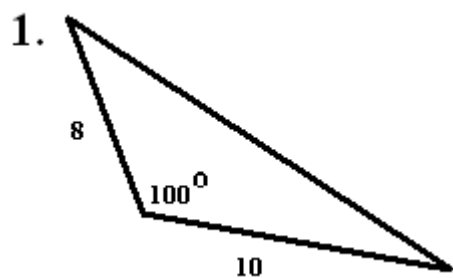
Which pairs of triangles are similar?



**Answers: 1, 3, and 4 are similar.**

## SAS Similarity Overhead Transparency

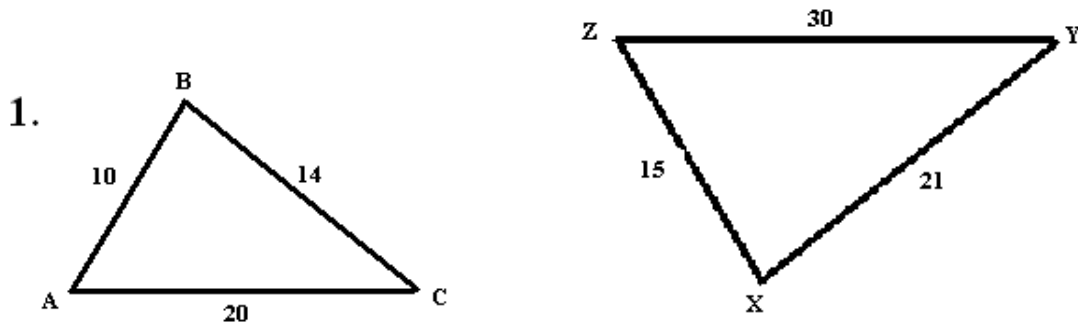
Which pairs of triangles are similar?



**Answers: Only 1 and 3 are similar.**



SSS Similarity Overhead Transparency



Is  $\triangle ABC \sim \triangle XYZ$ ?

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2.  $\triangle MNO$  has sides of lengths 4, 5, and 6.  $\triangle RST$  has sides of lengths 8, 9, and 10. Are the triangles similar?

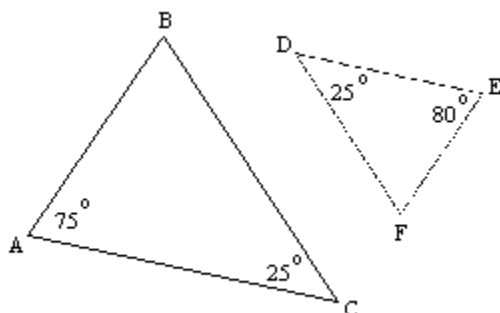
Answers: 1 – yes, 2 – no

# Identifying Similar Triangles

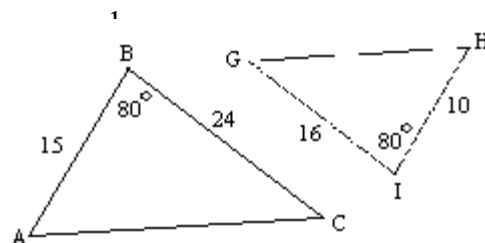
Name: \_\_\_\_\_  
Date: \_\_\_\_\_

Write a similarity statement for each pair of triangles and give the postulate or theorem that justifies your answer.

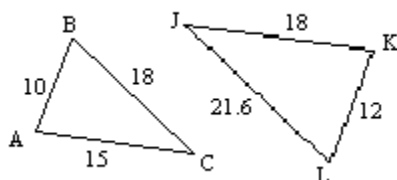
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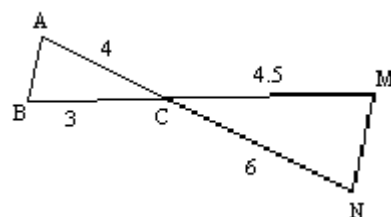
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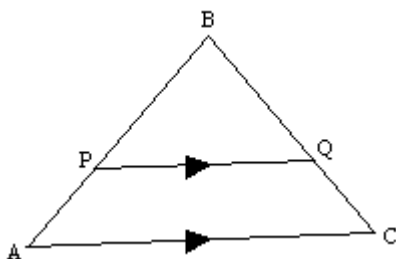
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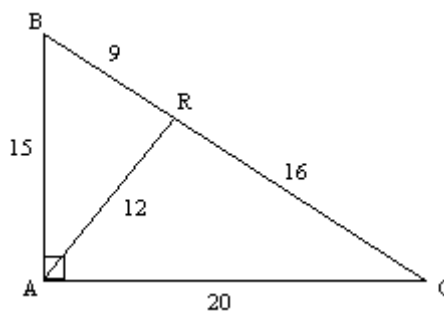
4.



5.



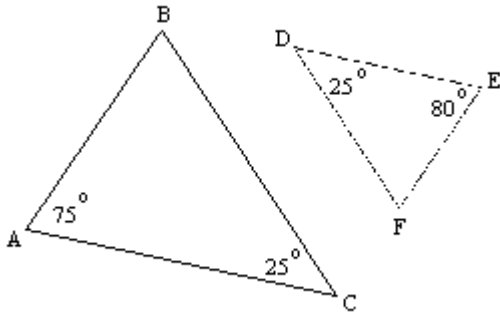
6.



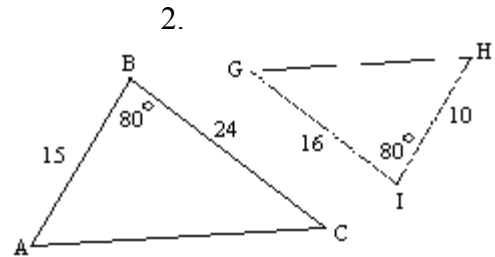
## Similarity Day 2: Identifying Similar Triangles Answer Key

Write a similarity statement for each pair of triangles and give the postulate or theorem that justifies your answer.

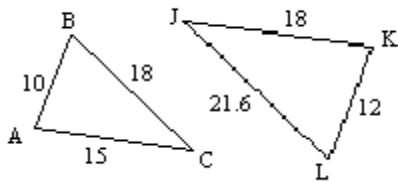
1.  $\triangle ABC \sim \triangle FED$  by AA



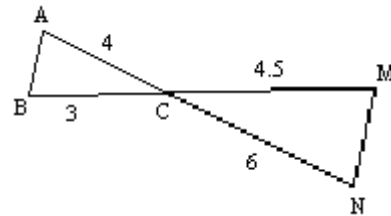
2.  $\triangle ABC \sim \triangle HIG$  by SAS



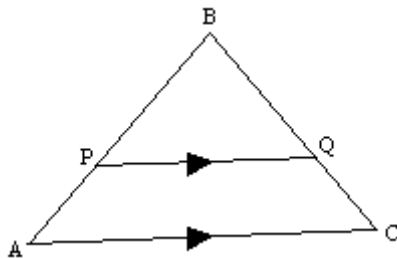
3.  $\triangle ABC \sim \triangle KLJ$  by SSS



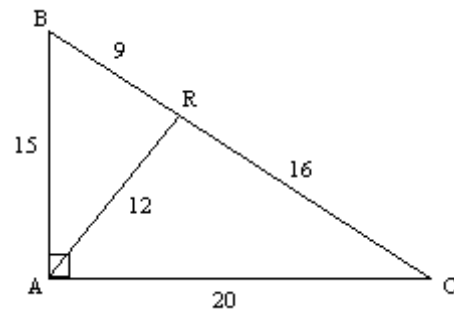
4.  $\triangle ABC \sim \triangle NMC$  by SAS



5.  $\triangle ABC \sim \triangle PBQ$  by AA



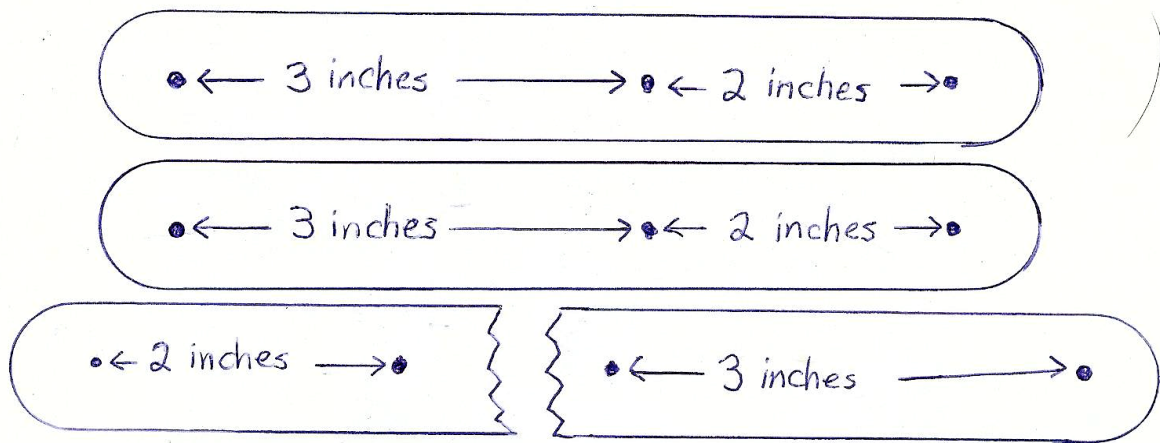
6.  $\triangle ABC \sim \triangle RAC \sim \triangle RBA$  by SSS



## Building and Using a Pantograph

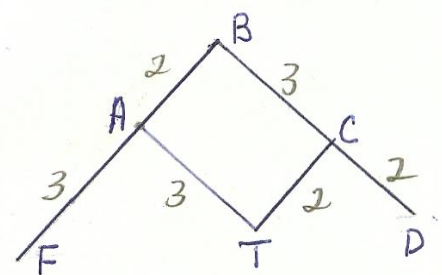
### Materials:

- 2 brass fasteners
- 3 one inch wood screws
- Mini suction cups
- One small marker
- 6 inch craft sticks (or emery boards) with holes drilled as shown using a 7/64 inch drill bit
- Masking tape for emergency repairs



### Directions for Assembling Pantograph

1. Arrange the four sticks to form the figure to the right. The whole sticks are represented by segments FB and DB, and the two smaller pieces are segments AT and CT. Align the holes in the sticks. ABCT should be a 2 inch by 3 inch parallelogram.
2. Fasten the sticks together with brass fasteners at points A and C.
3. Twist in screws at points B and T.
4. Attach suction cup at point F.
5. Insert the point of the marker at point D.
6. Adjust the screws and marker so that when placed on a flat surface, the marker touches down on the surface.
7. The pantograph is completed. Make sure it is connected loosely enough to bend; it should fold and unfold.



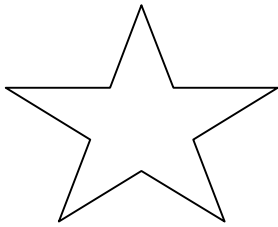
## Using your Pantograph

Name: \_\_\_\_\_

Date: \_\_\_\_\_

The pantograph traces an image using the point of the screw at point T. While point T traces the image, point D, the marker end, draws a magnified copy of the image. It is important to suction down point F—this part of the pantograph remains fixed. Tape your paper to the desk so it doesn't move.

Practice using your pantograph. Use it to make a magnified copy of the pictures below.



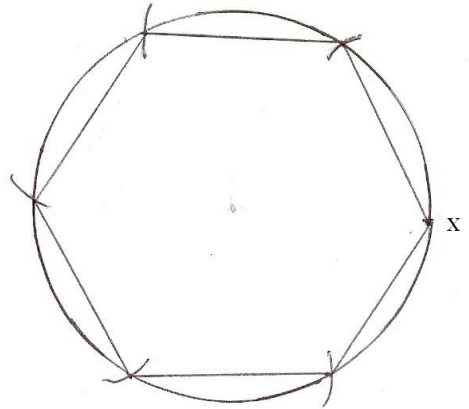
## Extension Activity--Constructing a Similar Hexagon Design

Name: \_\_\_\_\_

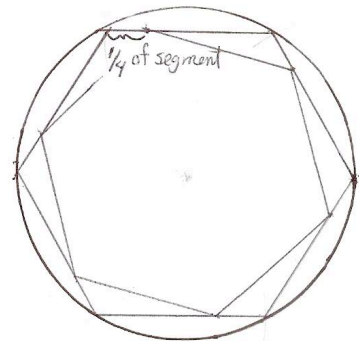
Date: \_\_\_\_\_

### Directions to Create the Design

1. Use a compass to draw a circle.  
A radius of about 3 inches works well. Mark point X on the circle, and, with the compass set to the radius of the circle, set the compass point on X and mark off congruent arcs along the circle. Connect these points. What regular polygon did you create?



2. Copy one side of your hexagon onto patty paper and fold it to divide the segment into 4 congruent segments. Mark the  $\frac{1}{4}$  point on each side, working in a clockwise direction (as shown). Connect these points. Is this new figure similar to the original? What is the scale factor of the first to the second hexagon?



3. Repeat this process at least four more times, each time copying the side length of the newest hexagon, dividing it into fourths, and marking the  $\frac{1}{4}$  point on each side. Do you see an optical illusion being formed?
4. Try a different shape with the same process. Do you think this process can be used to generate similar triangles? Rectangles? Are there polygons for which this process does not work?

#### Answers to Questions from building a pantograph

1. The scale factor should be  $\frac{3}{5}$ .
2. The scale factor is 3 to 5. The distance from the fixed point to trace point is 3 inches, and the distance from the fixed point to the drawing point is 5 inches.

#### Answers to Questions from Extension Activity

1. A regular hexagon. This can be verified by analyzing the angles that are formed from the radii drawn to the points.
2. Yes, this can be done with any convex polygon. If one begins with a nonconvex figure, there will be lots of overlapping, so the illusion of a spiral would not appear.

## Directions for Creating a Model of a Pantograph on CabriJr

Prepare this on CabriJr for the last lesson on similar triangles. Following these instructions will create a model of a pantograph on the TI84 screen which can be displayed using a viewscreen on an overhead projector.

1. Draw three segments to draw a triangle. Label the points A, D, and E as shown. Make sure the points are selected when you label them.



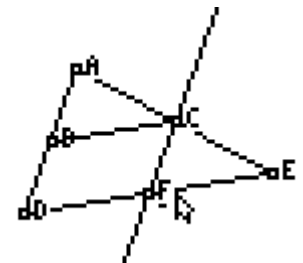
2. Construct the midpoints of segments AD and AE. Label the midpoints B and C. Make sure the points are selected when you label them.



3. Draw segment BC.



4. Construct a line through C that is parallel to segment AD. Draw (point on intersection) and label point F. Make sure the point is selected when you label it..



5. Draw segment CF. Hide the rest of line CF.



*When you grab point C and drag it around, the entire figure will move the way that a pantograph moves. Be sure to point out to the students that wherever point C is dragged, the smaller triangles and the large triangle remain similar.*



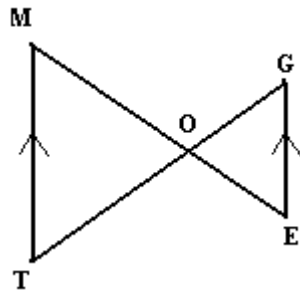
# Summative Assessment

Name: \_\_\_\_\_

Date: \_\_\_\_\_

1. For each pair of similar triangles, write AA, SAS, or SSS to describe why the triangles are similar and complete the similarity statement. If the triangles are not similar, explain why.

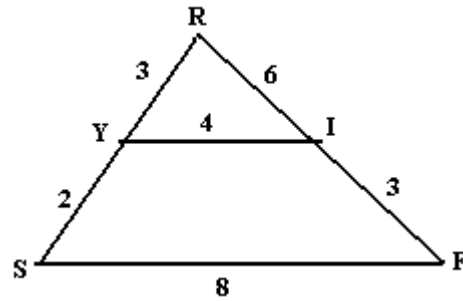
(a)



$\triangle MTO \sim \triangle GTE$  \_\_\_\_\_

Reason: \_\_\_\_\_

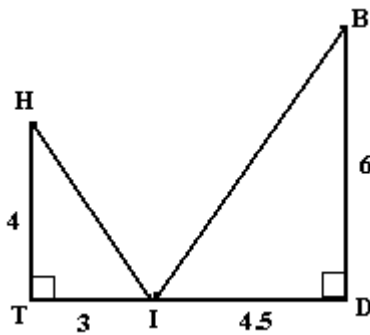
(b)



$\triangle RYI \sim \triangle RSF$  \_\_\_\_\_

Reason: \_\_\_\_\_

(c)

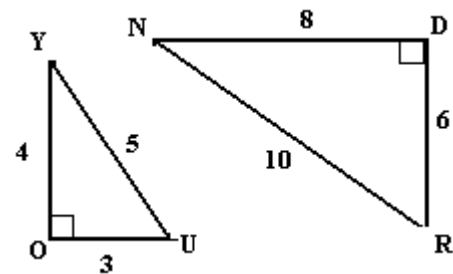


$\triangle HTI \sim \triangle BDI$  \_\_\_\_\_

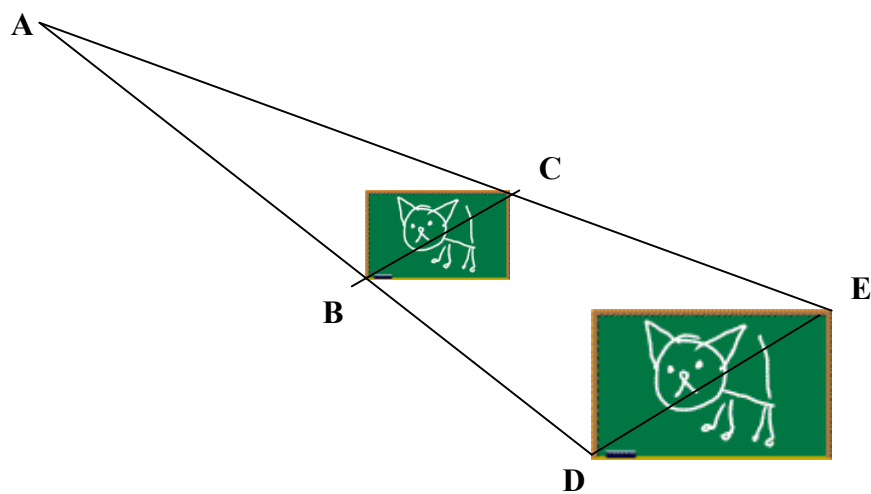
Reason: \_\_\_\_\_

2. Complete using the figure to the right.

$\triangle YOU \sim \triangle NDR$  because of the \_\_\_\_\_ similarity postulate. If  $m\angle O = 90^\circ$  and  $m\angle R = 50^\circ$ , then  $m\angle N =$  \_\_\_\_\_ and  $m\angle Y =$  \_\_\_\_\_.



3. For any position of the pantograph, the ratio of AB to AD is always equal to the ratio of AC to AE. Explain why this implies that  $\triangle ABC \sim \triangle ADE$ .



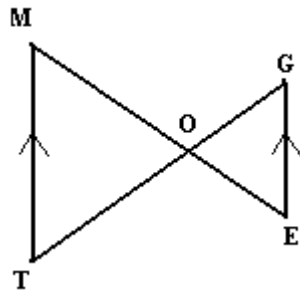
## Summative Assessment

Name: **Answer Key**

Date: \_\_\_\_\_

1. For each pair of similar triangles, write AA, SAS, or SSS to describe why the triangles are similar and complete the similarity statement. If the triangles are not similar, explain why.

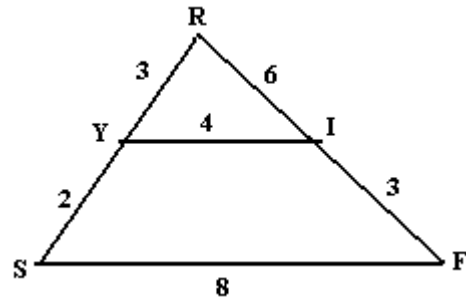
(a)



$\triangle MTO \sim \triangle GTE$

Reason: **AA**

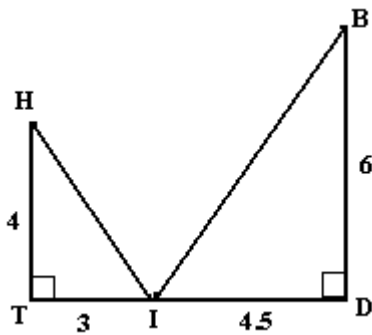
(b)



$\triangle RYI \sim \triangle RSF$

Reason: **Not Similar**

(c)

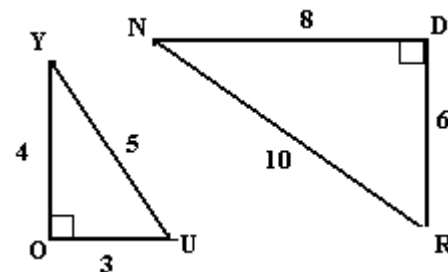


$\triangle HTI \sim \triangle BDI$

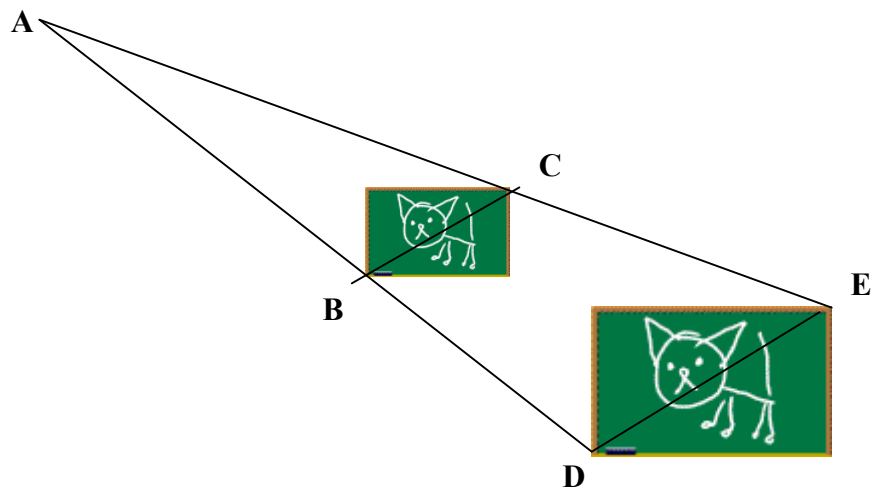
Reason: **SAS**

2. Complete using the figure to the right.

$\triangle YOY \sim \triangle NDR$  because of the **SSS or SAS** similarity postulate. If  $m\angle O = 90^\circ$  and  $m\angle R = 50^\circ$ , then  $m\angle N = 40$  and  $m\angle Y = 40$ .



3. For any position of the pantograph, the ratio of AB to AD is always equal to the ratio of AC to AE. Explain why this implies that  $\triangle ABC \sim \triangle ADE$ .



By the reflexive property of congruence,  $\angle A \cong \angle A$ , and since  $\frac{AC}{AE} = \frac{AB}{AD}$ , the triangles are similar by SAS.